Conference in Memory of Aurelio Carboni

Milano 24 - 26 June 2013

Conference Abstracts

Mal'cev categories vs protomodular categories

Dominique Bourn

This talk will be a survey of the graded relationships between Mal'cev and protomodular categories.

The symmetric topos in perspective: algebraic and geometric aspects

Marta Bunge

The symmetric topos $\Sigma(E)$ of a Grothendieck topos E is the classifier of Lawvere distributions on E with values in **Sets**. Lawvere [1] asked whether such a topos exists. A "set-theoretic" proof of its existence was given by myself [2] using forcing topologies in topos theory. The same result was then given an "algebraic proof" [3] by means of the finite limits completion at the level of sites. We proved therein that the symmetric KZ-monad on the 2-category A of locally presentable categories has the opposite of the 2-category R of Grothendieck toposes as its category of algebras. The 2-adjoint pair (Σ, U) where U is the forgetful from (R^{op}) to A, may also be viewed "geometrically", to wit, as a 2-adjoint pair (U^{op}, Σ^{op}), inducing a KZ-monad M on R. Unlike the algebraic situation, this 2-adjoint pair is not 2-monadic. The symmetric monad is an instance of what was called "completion monad" in [4] since, for any "locally faithful" completion monad M in a 2-category K, the M-algebras coincide with the "complete objects" in the sense of Street. I conjecture that, in such a context, every surjective 1-cell in K having an "M-adjoint" is of effective descent. I shall discuss a possible proof if this statement. Applications would be the known theorems about descent in the context of toposes and of locales, among possibly others.

[1] F.W.Lawere, Measures on toposes, Lecture at the Workshop on Categorical Methods in Geometry, Aarhus University, 1983.

[2] Marta Bunge, Cosheaves and distributions on toposes , Algebra Universalis 34 (1995) 469-484.

[3] Marta Bunge and Aurelio Carboni, The symmetric topos, J.Pure Appl. Algebra 105 (1995) 233-249.

[4] Marta Bunge and Jonathon Funk, Singular Coverings of Toposes, LNM 1980, Springer, 2006.

Monotone-light factorisation systems and torsion theories

Marino Gran

Given a torsion theory $(\mathbb{Y}, \mathbb{X})$ in an abelian category $\mathbb{C}$, the reflector $I : \mathbb{C} \to \mathbb{X}$ to the torsion-free subcategory $\mathbb{X}$ induces a reflective factorisation system $(\mathcal{E}, \mathcal{M})$ on $\mathbb{C}$. It was shown by A. Carboni, G. Janelidze, G.M. Kelly and R. Paré in [1] that $(\mathcal{E}, \mathcal{M})$ induces a monotone-light factorisation system $(\mathcal{E}', \mathcal{M}^*)$ by simultaneously stabilising $\mathcal{E}$ and localising $\mathcal{M}$ whenever the torsion theory is hereditary and any object in $\mathbb{C}$ is a quotient of an object in $\mathbb{X}$. In [2] we extend this result to arbitrary normal categories, and improve it also in the abelian case, where we show that the heredity assumption is not needed. It turns out that, under suitable assumptions, the reflective subcategory $\mathcal{M}^*$ of coverings in the category $\mathsf{Arr}(\mathbb{C})$ of arrows in $\mathbb{C}$ induces monotone-light factorisation systems in the category $\mathsf{Arr}^n(\mathbb{C})$ of n-fold arrows. Many examples of torsion theories where these results apply are then considered in the categories of abelian groups, groups, topological groups, commutative rings, and crossed modules.
This work is in collaboration with Tomas Everaert.

[1] A. Carboni, G. Janelidze, G.M. Kelly and R. Paré, On localization and stabilization of factorization systems, Appl. Categ. Structures 5, (1997) 1–58.
[2] T. Everaert and M. Gran, Monotone-light factorisation systems and torsion theories, accepted for publication in Bull. Sciences Mathématiques (2012).

Projective and affine aspects in categories

Marco Grandis

The papers cited below give characterizations of 'categories of affine spaces' (defined as slice categories of additive categories with kernels) and 'categories of projective spaces' (defined as quotients of abelian categories modulo a canonical congruence). Starting from these papers, we give a tentative discussion of projective and affine aspects in category theory, homological algebra and algebraic topology.

[1] A. Carboni, Categories of affine spaces, J. Pure Appl. Algebra 61 (1989), 243-250.
[2] A. Carboni - M. Grandis, Categories of projective spaces, J. Pure Appl. Algebra 110 (1996), 241-258.

Towards axiomatic description of categories of commutative algebras

George Janelidze

This is a joint work with Aurelio Carboni, which, sadly, could not be completed together. It consists of the published paper [1], many discussions between the authors, and several drafts of [2], the last of which was written by Aurelio in 2010. I shall briefly recall the results of [1] adding remarks suggested by the 'semi-abelian' developments, and then explain that the characterization is 'almost there'.

[1] A. Carboni and G. Janelidze, Smash product of pointed objects in lextensive categories, Journal of Pure and Applied Algebra 183, 2003, 27–43.

[2] A. Carboni and G. Janelidze, An axiomatic description of categories of commutative algebras.

The geometry (?) of realizability toposes

Peter T. Johnstone

Ten years ago, at the Workshop on Ramifications of Category Theory in Firenze, I gave a talk entitled 'A survey of realizability toposes', in which I pointed out the need for a better (categorical) understanding of 'the world in which realizability toposes live'. In particular, I raised the question whether the notion of geometric morphism has a significant role to play in our understanding of realizability toposes. For some time, the available evidence tended to suggest that the answer to this question was negative: for example, if λ and μ are Schönfinkel algebras and the cardinality of λ is greater than that of μ, then there are no geometric morphisms from the (ordinary) realizability topos over λ to that over μ. However, recent work – some of it coming from the 2013 PhD theses of Wouter Stekelenburg (Utrecht) and Jonas Frey (Paris), and some from my own ideas developing earlier work of Jaap van Oosten and Benno van den Berg – has tended to paint a more positive picture. The aim of my talk is to survey these recent developments.

Syntactical Presentations of Objective Abstract Generals and Axiomatizability, Distributors and Distributions

William F. Lawvere

One of Aurelio's important contributions was his Joint work with Marta Bunge showing that the space of distributions on a space exists, when "spaces" are interpreted to mean Grothendieck toposes and where the values of the distributions lie in a cocomplete category $\mathcal{V}$. By analogy with commutative algebra, this construction was called the "symmetric algebra" of $\mathcal{V}$, although the relevant products are automatically commutative.

T.B.A.

Giuseppe Rosolini

...

The Snail Lemma

Enrico M. Vitale

...

Racks and Tangled Circuits
Robert F. C. Walters

I will begin by tracing the history of work with Aurelio begun in 1980 on monoidal (bi)categories of relations leading up to the following recent developments:
In [1] the construction from a group G of a braided monoidal category TRel_G with a tangle algebra is described, thus yielding invariants for (tangled circuits including) blocked braids, but not resolving the question of whether blocked double torsions are always the blocked identity, as blocked four torsions are.
In recent work Davide Maglia investigated the minimal algebraic structure (instead of a group) needed for the construction TRel_G to be carried through, and was able to distinguish blocked two torsions from the blocked identity. Subsequently with Sabadini we have simplified the description of this minimal algebraic structure: G may be taken to be a rack (see [2,3]) with an additional bijective unary operation $(\ )^-$ satisfying $(g \rhd g^-)^- = g$, $g^- \rhd (g \rhd h) = h$, $g \rhd (g^- \rhd h) = h$, and $(g \rhd h^-) = (g \rhd h)^-$.
The crucial fact is that these axioms imply that $g^{----} = g$ but not $g^{--} = g$.

[1] R. Rosebrugh, N. Sabadini, R.F.C. Walters, Tangled Circuits, Theory and Applications of Categories,26, No. 27, 743–767, 2012
[2] Gavin Wraith, A Personal Story about Knots,
http://www.wra1th.plus.com/gcw/rants/math/Rack.html
[3] R.A. Fenn, C.P.Rourke, Racks and Links in Codimension Two, Journal of Knot Theory and its Ramifications 4, 343–406, 1992)

The waves of a total category and total distributivity.
Richard Wood
(Joint work with Francisco Marmolejo and Bob Rosebrugh)

Street and Walters defined a locally small category $\mathcal{K}$ to be *total(ly cocomplete)* if its Yoneda functor $Y : \mathcal{K} \longrightarrow \widehat{\mathcal{K}}$ has a left adjoint, X. We say that $\mathcal{K}$ is *totally distributive* if X has a left adjoint, W. It transpires that every total category admits a functor $W : \mathcal{K} \longrightarrow \widehat{\mathcal{K}}$ for which the associated hom-like functor, $\widetilde{\mathcal{K}}(-,-) : \mathcal{K}^{op} \times \mathcal{K} \longrightarrow \mathbf{set}$, has interesting properties.