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Higher Hopf formulae and fundamental group functors
in almost Barr-exact homological categories

In 1988, R. Brown and G. J. Ellis [1] gave Hopf formulae for the integral homology of a group using topological methods. G. Janelidze first recognized that these descriptions are deeply connected with the categorical Galois-theoretic notion of covering morphism. Using this perspective, T. Everaert, M. Gran and T. Van der Linden [3] proved that the same kind of formulae can describe the homology objects in a semi-abelian context when the coefficient functor is a Birkhoff reflector. It turns out that these homology objects coincide with the so called fundamental groups [4] arising in categorical Galois theory. I have developed this approach to homology and I have shown that one can work

- with a bigger class of reflectors: those which preserve pullbacks of split epimorphisms along regular epimorphisms;
- with a bigger class of categories: the homological categories in which every regular epimorphism is an effective descent morphism.

and still obtain some Hopf formulae. These formulae can also be refined when the reflector factors through a protoadditive functor (similarly to what was done in [2] for the first fundamental group functor within the semi-abelian context). This new results will be the main subject of my talk.

References:

[1] R. Brown and G. J. Ellis, *Hopf formulae for the higher homology of a group*, Bull. Lond. Math. Soc. 20 (2) (1988), 124–128.

[2] M. Duckerts, T. Everaert and M. Gran, *A description of the fundamental group in terms of commutators and closure operators*, J. Pure Appl. Algebra 216 (2012), 1837–1851.

[3] T. Everaert, M. Gran and T. Van der Linden, *Higher Hopf formulae for homology via Galois Theory*, Adv. Math. 5 (2008), 2231–2267.

[4] G. Janelidze, Galois groups, *abstract commutators and Hopf formula*, Appl. Categ. Structures 16 (2008), 653–668.