

On the characterisation of 3-permutable varieties

Diana Rodelo

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(with M. Gran)

We give a new characterisation of Goursat (=3-permutable) categories through a stability property for regular epimorphisms, a weaker version of a similar stability property which characterises Mal'tsev (=2-permutable) categories [1, 2]. On the other hand, it is well known that a variety of universal algebras is 3-permutable when its theory contains two ternary operations r and s satisfying the identities $r(x, y, y) = y$, $r(x, x, y) = s(x, y, y)$ and $s(x, x, y) = y$ [3]. We then show how this characterisation of 3-permutable varieties through the existence of ternary operations r and s can be directly obtained from a particular diagram in the category of free algebras, which is associated to the stability property mentioned above.

References

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- [2] M. Gran, D. Rodelo, *The Cuboid Lemma and Mal'tsev categories*, Appl. Cat. Struct. 22(5) 805-816 (2014)
- [3] J. Hagemann and A. Mitschke, *On n -permutable congruences*, Alg. Universalis 3, , 8-12 (1973)